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Virtual Manipulative for Mathematics Learning

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Abstract: *In order to succeed in mathematics students must develop an understanding of abstract concepts. Elementary teachers often use math manipulative to represent concretely the abstract concepts that students are learning and to connect these concepts to prior knowledge. Traditionally, teachers and students used concrete manipulative, however, in many contemporary classrooms teachers and students also use pictorial and virtual manipulative. This article will begin by defining, and providing examples and potential applications of concrete, pictorial, and virtual manipulative. Next, this article will present the theoretical foundations for teachers and students to use manipulative in mathematics education. Finally, this article will review the literature on the impacts of teachers and students using math manipulatives. The author hopes that this article reveals the factors and conditions that may contribute to educators' successes and struggles with using manipulative.*

Keywords: Mathematics, Mathematics Education, Manipulative, Representation Virtual Manipulative.

I. INTRODUCTION

Teaching for understanding in mathematics involves presenting the curriculum through multiple representations. Representations, which can be used in mathematics education, include physical (concrete), pictorial (static visual), and virtual (dynamic electronic) representations. In addition to the abstract (also referred to as symbolic) representation, math educators can use math manipulative to model multiple representations of math concepts. Each major math standards document published in the past 20 years has advocated for the use of manipulative. The fourth Mathematical Practice Standard in the Common Core State Standards for Mathematics (hereafter CCSSM) (National Governors Association Centre for Best Practices, Council of Chief State School Officers, 2015), "model with mathematics" highlights students' use of models to communicate their thinking when solving math problems. Teachers striving to develop students' capacity to "model with mathematics" should explicitly make connections between real-world scenarios and mathematical representations of those scenarios during their instruction. Moreover, The National Council of Teachers of Mathematics (NCTM, 2000) Principles and Standards for School Mathematics (PSSM) encourage teachers and students to use multiple representations during mathematics instruction. The PSSM states that all students should "create and use representations to organize, record, and communicate mathematical ideas; select, apply, and translate among mathematical representations to solve problems; use representations to model and interpret physical, social, and mathematical phenomena" . The primary purpose of this research review is to survey the literature regarding the effectiveness of using physical, pictorial, and virtual manipulative at enhancing students' conceptual understanding of abstract concepts. Prior to evaluating the efficacy of physical, pictorial, and virtual manipulative, this article will define each form as well as their potential applications. Next, this article will examine the theoretical foundation supporting the use of manipulative. Subsequently, this article will present the findings of contemporary research studies on the advantages and disadvantages of the use of manipulative. The author hopes that this research review uncovers the factors and conditions that contribute to educators' successes and struggles with using manipulative.

Many students with learning disabilities (LD) demonstrate poor conceptual understanding of fractions, limiting their ability to solve more advanced problem-solving tasks, including ratios, rates, and proportions. When working with students with LD who have individualized education programs in the area of mathematics, recent research supports the use of interactive visual models called virtual manipulative ,because of the ability to enable the design features to adjust instructional demands, scaffold mathematical content, and increase the amount of practice opportunities, all of which can be incorporated into intensified instruction for students with LD. Initially, virtual manipulative were defined as web

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based images on a computer monitor that permitted students to manipulate a visual model as if it were three-dimensional. Previous syntheses and meta analyses of methods of teaching fractions to students with LD recommended the use of visual models ,because of their potential in helping students to better understand the meaning and relationship of fractions through relevant diagrams (e.g., part-whole relationship is represented with a divided circle, multiplication of fractions is represented with a rectangular area model). Through technology, students can access virtual manipulative on mobile devices such as iPads in addition to web based computer applications. This capability to easily access virtual manipulative allows students opportunities to engage with the visual models as part of their intervention on various concepts and skills related to fractions. For example, teachers can use a variety of virtual visual models such as fraction bars, area models, and number lines to teach equivalent fractions and addition, subtraction, multiplication, and division of fractions.

This article discusses

1. Technology trends in teaching mathematics to students with LD,
2. Virtual manipulative as instructional mathematical tools for use in the classroom,
3. The benefits of using virtual manipulative, and
4. Potential challenges with using virtual manipulative for instructional purposes.

II. VIRTUAL MANIPULATIVE AS INSTRUCTIONAL MATHEMATICAL

Tools In responding to the needs of "web natives" who are adept at accessing and manipulating technology devices, virtual manipulative provide a variety of classroom opportunities. Teachers can implement virtual manipulative as a form of interactive visual models to teach the concept of fractions during their lessons. When virtual manipulative are presented in the form of games and quizzes, teachers can use them to check for understanding of fraction concepts and skills. Table 1 summarizes the features of web-based and i-Pad applications of virtual manipulative for selected products. Teachers can integrate the use of visual manipulative tools into their instruction by demonstrating how the models can be used to teach new concepts, such as fractions, and providing students opportunities to engage in interactive activities during guided and/or independent practice. Several web-based virtual manipulative applications provide build your own fractions options, whereby teachers can create their own fraction problems and control virtual manipulative as visual model tools for instruction. Because virtual manipulative provide a visual model tool, teachers must be able to guide students in how to use them and make the connection between visual models and what they represent.

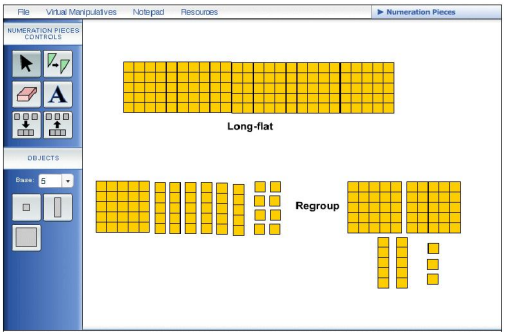


Fig. 1

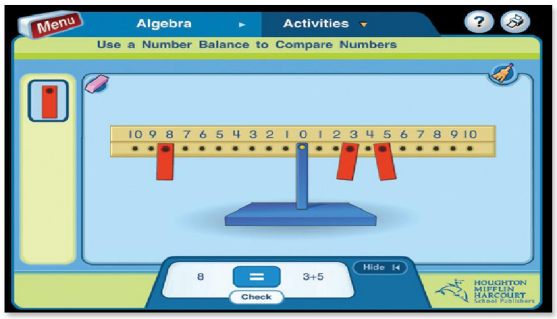


Fig .2

The Common Core State Standards for Mathematics (NGAC/CCSSO, 2010) include the recommendation for using various tools, such as virtual manipulative. This recommendation can be translated into instruction by having students manipulate visual models and importantly explain their models in relation to mathematical equations to be solved. For example, when solving fraction problems, teachers should demonstrate how to manipulate quantities that illustrate the meaning of the fraction number (i.e., numerator and denominator quantities). Students should be expected to talk about how they are using the visual model tools to show their work while solving problems. Free and available web-based applications offer opportunities for teachers and students to implement virtual manipulative (i.e., visual model tools)

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during mathematics instruction. For example, using Illuminations (http://illuminations.nctm. org/Games-Puzzles.aspx) or Conceptual Math (http://www. conceptuamath.com), teachers can provide instruction in fractions during whole-class and small-group instruction. These web-based applications offer various visual model tools (e.g., pattern block, number line, vertical bar, dots) for each relevant fraction concept (e.g., equivalent fractions, addition, subtraction, multiplication, division, word problems). Using interactive whiteboards, teachers can demonstrate how to connect virtual manipulative to each fraction concept. In addition, using iPad applications (e.g., ABCya. com's Virtual Manipulative!, Braining camp's Dividing Fractions), teachers can help individual students explore fractions using their own mobile devices. After the teacher has introduced concepts and skills related to equivalent fractions or dividing fractions, students can individually explore fraction visual models using iPad apps. Students can drag pieces of circles or fraction bars into the workspaces. By comparing the size of sliced circles or length of fractions bars, students can compare equivalent fractions. In particular, Virtual Manipulative Provides note functions through which students can draw any other fraction diagrams and write equations if needed.

III. Challenges and Solutions With Virtual Manipulative

The use of virtual manipulative for teaching fractions with students with LD presents four major challenges. First, in general, most teachers have not been trained on how to use and select the right device to meet individual students' needs (McMahon & Walker, 2014). To alleviate the challenge of incorporating virtual manipulative into their instruction, teachers can use video tutorials and instruction provided in virtual manipulative websites and applications. For websites and applications that provide video tutorial options. Using the tutorials and instruction, teachers can easily follow how to implement the virtual manipulative in their lesson plans. However, only some of the websites and applications provide the tutorials; virtual manipulative websites and applications developers should consider providing the tutorials for users. Second, although the video tutorials can be useful for teachers, they are insufficient. Many researchers have warned that virtual manipulative in and of themselves do not guarantee that students are developing mathematical conceptual understanding (Moyer-Packenham & Westenskow, 2013).

When using virtual manipulative, teachers must check students' ability to use virtual manipulative to accurately connect their visual models to mathematics concepts (McMahon & Walker, 2014). During this process, students should be encouraged to verbalize their mathematical thinking and justify their representations of problem situations (Hunt, 2014). Teachers should monitor students' actions of linking two dynamic visual models on the screen or touch pad by asking reflective questions such as these: "What does the virtual fraction bar represent in the given situation? What happened to the graph when you changed the spinner? How can we define the mathematical relationship between the two virtual area models on the screen?" (Hunt, 2014; Moyer-Packenham & Westenskow, 2013). In this way, students should be shown a visual representation of a mathematical concept and asked to explain it; or, in the reverse situation, given problems such as equivalent fractions, students should be able to represent this concept using virtual manipulative (Dougherty, D. P. Bryant, B. R. Bryant, Darrough, & P fannenstiel, 2015).

Third, teachers must employ thoughtful selection criteria to choose from a vast array of visual models offered on websites and as applications. Much like software, there is a need to consider instructional design features particularly as they relate to the instructional needs of students with LD. For instance, teachers can utilize a rubric when evaluating and selecting web-based or iPad applications. Ok, Kim, Kang, and B. R. Bryant (2016) developed a rubric form that teachers can use to evaluate instructional applications for teaching students with LD. The rubric form includes three sections: (a) identifying information such as content area, objectives, and type of application, (b) evaluating instructional features such as examples, progress monitoring, and feedback, and (c) grading options. Using the rubric can be helpful for teachers to identify and select high-quality virtual manipulative applications. Finally, in recognizing the need for intensive instruction for students with LD, teachers should embed the use of virtual visual models during guided practice of mathematical concepts and skills as part of "intensive, strategic, and explicit" instruction (B. R. Bryant et al., 2016, p. 9). Moreover, because not all students respond to virtual visual models in the same way, teachers will likely need to modify the instructional delivery process. For example, teachers can scaffold instruction by first providing physical manipulative and then applying virtual manipulative in the extension of the previously taught

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concepts. Bryant et Instead of using all of the multimedia options provided by virtual manipulative websites and applications, teachers can carefully select mathematical inputs and symbols from the multiple options to incorporate as a means for intensifying instruction (Mayer & Moreno, 2003).

IV. Struggles Encountered Using Manipulative

However, there are also studies that do not support the use of manipulative. These studies focus on students not making connections between multiple representations, learning procedures rather than understanding concepts, inability to transfer knowledge to new applications, and learners having fun with the manipulative and essentially utilizing it as a toy.

V. Benefits of Virtual Manipulative

5.1 Cognitive Benefits

Virtual manipulative are powerful cognitive tools for learners because they "constrain the users' actions on the mathematical object in the virtual environment, directing the user to focus on the mathematics in the environment; they react to user input with visual and verbal/symbolic feedback showing the user the results of their actions on the object; and, they enforce mathematical rules of behaviour". Virtual manipulative combine several representations in a manner that increases student's capability to translate among multiple representational systems and thereby influences a student's ability to model and understand mathematical concepts. In comparison to the Dual Coding Theory, virtual manipulative are primarily visual representations that activate multiple systems of codes which can facilitate greater access to memory. In a study conducted by virtual manipulative were found to allow for discovery learning, allowed students to make their own individual conjectures, and encouraged students to see he mathematical relationships. In a specific example using a Fraction Addition applet, the virtual manipulative linked symbolic and iconic modes, and helped to deter common fraction errors .When it came to a virtual manipulative game, students were able to create student-to-student communication of mathematical ideas and apply previously learned concepts to new mathematical situations "Both Dual coding theory and Multimedia principles support the notion that when learners are presented with visual and verbal codes, the effects of the multimedia instruction and students' recall of information are increased" .The overall capability of virtual manipulative allows for cognitive experiences that students would not otherwise achieve in a traditional curriculum instructional classroom.

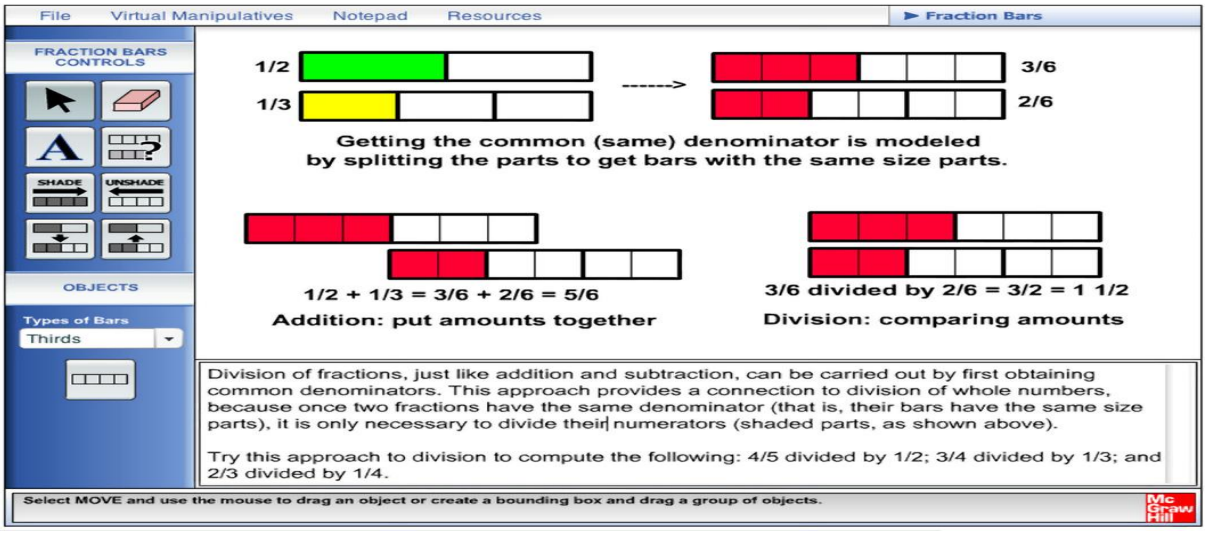


Fig 3.

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5.2 Learning Gains

Several studies have been conducted to determine the overall effect of virtual manipulative on student's overall achievement and learning gains. It has been found the low functioning groups of students benefit the most from virtual manipulative. When focusing on students with Autism spectrum disorder, all students in the study "demonstrated greater accuracy and faster independence with the virtual manipulative". Moyer and Packenham found that virtual manipulative resulted in overall gain but that the low functioning groups showed a significant statistical gain while the average and high achieving groups experienced only a numerical gain (2012). Suh et al (2005) discussed the fact that high functioning students would work the problems out on paper and then check their answers with the virtual manipulative while the low functioning groups benefited the most from the actual use of the virtual tools. Moyer et al (2014) discussed that recent studies on student gains have shown that demographic indicators of success are less likely to predict overall gains when students use virtual manipulative. Despite socioeconomic status, gender, race, and English language learner status, virtual manipulative create an equalizing effect on student achievement (Moyer et al, 2014, p. 121). Twelve distinct studies have shown that virtual manipulative do in fact support and increase student achievement and overall academic gains (Moyer et al, 2014). Virtual manipulative are not enough on their own but when used in combination with other methods of instruction, show the greatest gains in student achievement.

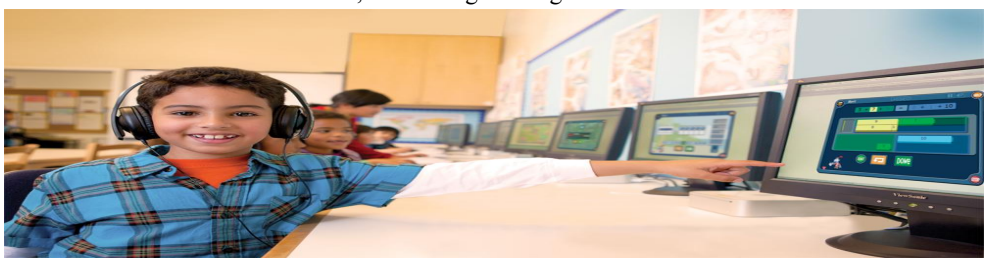


Fig .4

5.3 Disadvantage

When discussing the disadvantages of virtual manipulative, it is first important to cover the disadvantages that are specifically linked to technology use. The challenge presented by technology is that it is most prevalent in the area of teachers' proficiency levels and their ability to properly apply technology practices in the salesroom .Many teachers with low technology proficiency levels fear that they will "look stupid" in front of their students thus lowering the professionalism of the classroom. There is also a serious lack of professional development available for teachers in order to increase proficiency levels and to become more familiar with the tools available to them.

1. To become more specific to virtual manipulatives, many teachers are unaware of the "capabilities of virtual manipulative and do not currently use them in lessons during regular mathematics instruction" (Suh et al, 2005, p. 2). There is also the fact that the true definition of virtual manipulative is not widely known. Thus "Teachers who would provide opportunities for their students to construct mathematical knowledge must learn to distinguish between what is truly a virtual manipulative and what is merely 'virtual'" (Moyer et al, 2002, p. 373).
2. Teachers may also not "embrace technology because they believe the internet is unreliable or that a particular web site did not prove to be worthwhile, or they may be limited by the lack of technology resources and support" (Suh et al, 2005, p. 11). Limited technological resources are a highly limiting factor in the use of virtual manipulative. In order for them to truly display their learning and cognitive benefits, they must be used by every students and not in a mere teacher demonstration, therefore, this lack of available technology is a true disadvantage to students access to virtual manipulative. Virtual manipulative are also not designed in a way to support collaborative learning which an especially large problem is for the socio-cultural constructivist educators (Durmus et al, 2006). "Employing manipulative in a class is not straightforward and good employment requires carefully defining the role of the teacher and the aims and the potential tasks involved"

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(Durmus et al, 2006). This could require a lot of work on the part of the educator making several stray from using virtual manipulative all together. There are several misconceptions about virtual manipulative such as: manipulative do not simplify the learning of concepts, the more manipulatives used the better, and that they are better used in primary grades then at the secondary level (Durmus et al, 2006). These facts are just not true and it takes a dedicated educator to decipher what is beneficial and what is not. Despite the disadvantages listed here, it is safe to say that if one can overcome the initial hesitation to the introduction of technology.

According to Diene's Perceptual Variability Principle (1971), learners benefit from their teachers exposing them to the same concept multiple times using a variety of materials. Dienes believed that learners would develop a better understanding of the concept by making connections between these diverse experiences. According to Diene's Mathematical Variability Principle (1971), teachers who present multiple examples of a concept to their students where they hold the relevant variables constant and systematically alter the irrelevant variables will be more likely to have students who are able to generalize about the concept.

According to Diene's Constructivity Principle (1971), teachers should always provide their students with opportunities to work with concrete forms of concepts before they expect their students to analyze the abstract forms of the concepts. There is a close alignment between most of Bruner's writings and those of Piaget and Dienes. However, Bruner departed from Piaget in his belief that readiness for learning depends more upon effective instruction and learning experiences than when the learner reaches a particular age. Bruner (1966) asserts, "Any subject can be taught effectively in some intellectually honest form to any child at any stage of development" (p. 33). Bruner (1966) suggested that learners understanding of a concept progresses through three levels of sophistication of thought. Bruner refers to learning through concrete experiences as "enactive,"

VI. Concluding Thoughts

By promoting an interactive learning environment, teachers can help students with LD engage in their learning of mathematics. The use of virtual manipulatives is a viable tool for teaching fraction concepts and skills to students with LD. Virtual manipulatives can be integrated into teacher-delivered instruction as a visual model tool to facilitate students' conceptual understanding. Also, they can be used as a form of checking for understanding or as games and quizzes during differentiated instruction or remediation (Regan, Berkeley, Hughes, & Kirby, 2014). Regardless of the instructional situation, teachers must monitor students to make sure they fully understand what they are doing with the tools and, as a result, have a meaningful learning experience.

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